Sec. 11.5A Complex Numbers

Division Theorem:

$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)}$ or $f(x) = q(x)g(x) + r(x)$ where $r(x)$ is the remainder, $q(x)$ is the quotient and $g(x)$ is the divisor.

Remainder Theorem: If $f(x)$ is divided by $x - c$, then the remainder of $f(x)$ is $f(c)$.

Ex: Find the remainder if $f(x) = 4x^3 - 3x^2 - 8x + 4$ is divided by $x - 3$ and $x + 2$.

$x-3$: $f(3) = 4(3)^3 - 3(3)^2 - 8(3) + 4$
$= 108 - 27 - 24 + 4$
$= 61$

REMAINDER = 61

$x+2$: $f(-2) = 4(-2)^3 - 3(-2)^2 - 8(-2) + 4$
$= -32 - 12 + 16 + 4$
$= -24$

REMAINDER = -24

Factor Theorem:

If $f(c) = 0$, then $x - c$ is a factor of $f(x)$.
If $x - c$ is a factor of $f(x)$ then $f(c) = 0$.

Ex: Does $2x^3 - x^2 + 2x - 3$ have the factor $x - 1$ or the factor $x + 2$?

$x-1$: $f(1) = 2(1)^3 - (1)^2 + 2(1) - 3$
$= 2 - 1 + 2 - 3$
$= 0$

$(x-1)$ is a factor

$x+2$: $f(-2) = 2(-2)^3 - (-2)^2 + 2(-2) - 3$
$= -16 - 4 - 4 - 3$
$= -27$

$(x+2)$ is not a factor

Intermediate Value Theorem: Let f denote a continuous function. If $a < b$ and if $f(a)$ and $f(b)$ are of opposite sign, then f has at least one zero between a and b.

Ex: Use the Intermediate Value Theorem to show that there would be a zero on the given interval. Then approximate the zero the nearest hundredth.

a. $f(x) = 8x^4 - 2x^2 + 5x - 1$ on [0, 1]

$f(0) = 8(0)^4 - 2(0)^2 + 5(0) - 1$
$= 0 - 0 + 0 - 1$
$= -1$

$f(1) = 8(1)^4 - 2(1)^2 + 5(1) - 1$
$= 8 - 2 + 5 - 1$
$= 10$

SIGN CHANGED, ZERO BETWEEN $x = 0$ and $x = 1$

$\boxed{X = .22}$ From Graph

b. $f(x) = 2x^3 + 6x^2 - 8x + 2$ on [-5, -4]

$f(-5) = 2(-5)^3 + 6(-5)^2 - 8(-5) + 2$
$= -250 + 150 + 40 + 2$
$= -58$

$f(-4) = 2(-4)^3 + 6(-4)^2 - 8(-4) + 2$
$= -128 + 96 + 32 + 2$
$= 2$

SIGN CHANGED, ZERO BETWEEN $X = -5$ and $X = -4$

$\boxed{X = -4.05}$ From GRAPH

Complex Number: Form of $a + bi$ or $a - bi$ where a and b are real numbers and $i = \sqrt{-1}$ and $i^2 = -1$. Note: If $b = 0$, then the complex number is real.

Ex: 1.) $(4-3i)-(7-5i)$ 2.) $(5+3i)(2-i)$ 3.) $(x-(3+2i))(x-(3-2i))$

1.)
$4-3i-7+5i$
$-3+2i$

2.)
$10-5i+6i-3i^2$
$10+i-3(-1)$
$10+i+3$
$13+i$

3.)
$(x-3-2i)(x-3+2i)$
$x^2-3x+2ix-3x+9-6i-2ix+6i-4i^2$
$x^2-6x+9-4(-1)$
$x^2-6x+9+4$
$x^2-6x+13$

If $r_1 = a+bi$ and $r_2 = a-bi$, when the linear factors $(x-r_1)$ and $(x-r_2)$ are multiplied

$$(x-r_1)(x-r_2) = x^2 - 2ax + a^2 + b^2$$

Complex Conjugate Pairs Theorem: If $a + bi$ is a zero of f, then $a - bi$ will also be a zero of f.

-A polynomial f of odd degree with real coefficients has at least one zero.

Ex: If $-3+4i$ is a zero of $f(x)$, list two factors of $f(x)$.

FACTORS: $(x-(-3+4i))(x-(-3-4i))$

Fundamental Theorem of Algebra: Every complex function $f(x)$ of degree n greater than or equal to 1 has at least one complex zero.

Every complex polynomial function $f(x) = a_n(x-r_1)(x-r_2)\ldots(x-r_n)$ where a_n, r_1, r_2, $\ldots r_n$ are complex numbers. That is every complex polynomial function of degree $n \geq 1$ has exactly n (not necessarily distinct) zeros.

Ex: Solve $x^2 - 6x + 10 = 0$

CANNOT BE FACTORED - NO X-INTERCEPTS
USE QUADRATIC FORMULA TO FIND COMPLEX ZEROS

$\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\dfrac{6 \pm \sqrt{(-6)^2-4(10)}}{2}$

$\dfrac{6 \pm \sqrt{36-40}}{2}$

$\dfrac{6 \pm \sqrt{-4}}{2}$

$\dfrac{6 \pm \sqrt{4}\sqrt{-1}}{2}$

$\dfrac{6 \pm 2i}{2}$

$\boxed{3 \pm i}$

$x = 3+i$
$x = 3-i$

The power of the function will tell you how many total zeros there are. Find any real zeros by graphing. Divide the polynomial by the related factor(s) and then solve the quotient to find the remaining non-real, complex zeros. The quotient can be solved using the quadratic formula or by simply solving for x if there in no b term.

Ex: Find a polynomial of degree 4 whose coefficients are real numbers and that has the zeros 1, 1, and $-4+i$. Graph to verify your result.

$(x-1)(x-1)(x-(-4+i))(x-(-4-i))$

$(x^2-2x+1)(x^2+8x+17)$

$x^4+8x^3+17x^2-2x^3-16x^2-34x+x^2+8x+17$

$x^4+6x^3+2x^2-26x+17$

$a=-4 \quad b=1$

$x^2-2ax+a^2+b^2$

$x^2-2(-4)x+(-4)^2+1^2$

$x^2+8x+16+1$

$x^2+8x+17$

Ex: Find the complex zeros of the polynomial function $f(x)$.
$f(x)=3x^4+5x^3+25x^2+45x-18$. HINT: Find real zeros first!!

REAL ZEROS: $x=-2 \quad x=\frac{1}{3}$

REAL FACTORS: $(x+2)(x-\frac{1}{3})$

$x^2-\frac{1}{3}x+2x-\frac{2}{3}$

$x^2+\frac{5}{3}x-\frac{2}{3}$

$$
\begin{array}{r|l}
 & 3x^2+27 \\
x^2+\frac{5}{3}x-\frac{2}{3} & 3x^4+5x^3+25x^2+45x-18 \\
 & -\;(3x^4+5x^3-2x^2) \\
\hline
 & 27x^2+45x-18 \\
 & -\;(27x^2+45x-18) \\
\hline
 & 0
\end{array}
$$

COMPLEX ZEROS: $3x^2+27=0$

$\frac{3x^2}{3}=\frac{-27}{3}$

$x^2=-9$

$x=\pm\sqrt{-9}$

$\boxed{x=\pm 3i}$

If there is a "b" term, must use quadratic formula!

HOMEWORK: p 1045 #3, 11, 15, 41, 49, 50, 59
p 257 #4-10, 64-68, 78 (Even)
p 264 #8, 10-13, 27, 28, 31, 32